

Scaling behaviours of the p_T spectra for identified hadrons in pp collisions

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Abstract.

We extend the scaling behaviour observed in the inclusive charged hadron transverse momentum (p_T) distributions to the p_T spectra of pions, kaons and protons produced in proton-proton (pp) collisions with center of mass energies (\sqrt{s}) at 0.9, 2.76 and 7 TeV. This scaling behaviour arises when a linear transformation, $p_T \rightarrow p_T/K$, is applied on the pion, kaon or proton p_T spectra. The scaling parameter K depends on \sqrt{s} and is determined by a new method, the quality factor method, which does not rely on the shape of the scaling function. We argue that the pions, kaons and protons originate from different distributions of clusters which are formed by strings overlapping, and the scaling behaviours of these identified particles p_T spectra could be understood with the colour string percolation model in a quantitative way simultaneously.

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1. Introduction

One of the most important observations in high energy collisions is the p_T spectra for different species of final state particles. From the spectra, we can learn a lot about the regularities of the particle productions.

In many studies, searching for a scaling behaviour of the p_T spectra is helpful to reveal these regularities. In [1], the authors showed a scaling behaviour in the pion p_T spectra with different collision centralities at midrapidity in Au+Au collisions at the Relativistic Heavy Ion Collider (RHIC). This scaling behaviour was extended to noncentral regions in Au+Au and d+Au collisions [2]. Similar scaling behaviours were found in the proton and anti-proton p_T spectra with different collision centralities at midrapidity in Au+Au collisions at RHIC [3].

Recently, we observed a scaling behaviour in the p_T spectra of inclusive charged hadrons in proton-proton (proton-antiproton) collisions at $\sqrt{s} = 0.9, 2.76$ and 7 (0.63, 1.8 and 1.96) TeV when a linear transformation, $p_T \rightarrow z = p_T/K$, was applied on these spectra [4]. Here the scaling parameter K depends on the collision energy \sqrt{s} . This scaling behaviour was explained by the colour string percolation model in a qualitative way. In this paper, we will extend this scaling behaviour to the p_T spectra of pions, kaons and protons produced in pp collisions at 0.9, 2.76 and 7 TeV. A new method, the quality factor method [5, 6], will be adopted in the searching for the scaling parameter K . Compared with the method utilized in [4], this method does not rely on the shape of the scaling function, thus it is more robust. We will argue that the pions, kaons and protons originate from different distributions of clusters which are formed by strings overlapping, and the colour string percolation model can describe the scaling behaviours of the identified p_T spectra not only in a qualitative way but also in a quantitative way simultaneously. As a similar geometrical scaling behaviour was suggested when the p_T spectra of identified particles in pp collisions at 0.9, 2.76 and 7 TeV were presented in terms of a scaling variable $\tau = p_T/Q_0(p_T/\sqrt{s})^{\lambda/2}$, where $Q_0 = 1$ GeV/c and λ is around 0.27 [7], we would like to compare the scaling behaviours presented in z and τ .

The organization of the paper is as follows. In section 2, we will illustrate the procedure to search for the scaling behaviours in the p_T spectra of identified particles. In section 3, the scaling behaviours of pions, kaons and protons will be described. Section 4 shows the comparison between the scaling behaviours presented in the variables z and τ . In section 5, we will use the colour string percolation model to explain the scaling behaviours of the identified particles in a quantitative way. Finally, the conclusion is made in section 6.

2. Method to search for the scaling behaviours

As done in [4], we will search for the scaling behaviours of the identified pion, kaon and proton p_T spectra with the following steps. Let us take the pion p_T spectra as an example, we define a scaling variable, $z = p_T/K$, and a scaled p_T spectra,

$\Phi(z) = A \cdot (2\pi p_T)^{-1} d^2N/dp_T dy|_{p_T=Kz}$. Here the parameters K and A depend on the collision energy. By choosing proper K and A , the data points of the pion p_T spectra in pp collisions at 0.9, 2.76 and 7 TeV will migrate into one curve. As a convention, we usually set both K and A of the highest energy collisions to be 1. However, the p_T coverages of the pion spectra at 0.9 and 7 TeV are much smaller than the coverage at 2.76 TeV. In order to make the scaling function $\Phi(z)$ to describe the pion spectra in the large p_T region faithfully, we choose both K and A for the collisions at 2.76 TeV as 1. With different choices of K and A , the scaling functions $\Phi(z)$ are different. In order to get rid of this arbitrariness, we introduce another scaling variable, $u = z/\langle z \rangle = p_T/\langle p_T \rangle$, and the corresponding normalized scaling function $\Psi(u) = \langle z \rangle^2 \Phi(\langle z \rangle u) / \int_0^\infty \Phi(z) z dz$. Here $\langle z \rangle = \int_0^\infty z \Phi(z) z dz / \int_0^\infty \Phi(z) z dz$. The ways to search for the scaling behaviours in the p_T spectra of kaons and protons are identical to the one for pions.

3. Scaling behaviours of identified pions, kaons and protons

In this paper, the p_T spectra of identified pions, kaons and protons in pp collisions at 0.9, 2.76 and 7 TeV were published by the ALICE collaboration [8, 9, 10]. Here the identified pion, kaon and proton p_T spectra refer to the spectra of $(\pi^+ + \pi^-)/2$, $(K^+ + K^-)/2$ and $(p + \bar{p})/2$. The data at 2.76 TeV cover a pion p_T range of 0.1-20.0 GeV/c, which is much wider than the ones at 0.9 and 7 TeV, 0.1-2.6 GeV/c and 0.1-3.0 GeV/c. Similar results are obtained in the comparison among the kaon or proton p_T ranges at 0.9, 2.76 and 7 TeV. As described in section 2, both the parameters K and A at 2.76 TeV are set as 1, thus the scaling function $\Phi(z)$ is nothing but the p_T spectra of pions, kaons or protons at this energy. In [4], $\Phi(z)$ is written in the Tsallis form, in which the mass effect of the charged hadron is ignored. In this work, since the threshold of the p_T range is 0.1 (0.1 and 0.1) GeV/c at 2.76 TeV [10], which is below the mass of pions (kaons and protons) 0.14 (0.494 and 0.938) GeV/c² [11], we write the scaling function $\Phi(z)$ for pions, kaons or protons with the modified Tsallis form

$$\Phi(z) = C_q \left[1 - (1 - q) \frac{\sqrt{m^2 + z^2} - m}{z_0} \right]^{\frac{1}{1-q}}, \quad (1)$$

where C_q , q and z_0 are free parameters, m is the mass of the particle species, and $1 - q$ is a measure of the non-extensivity. These free parameters are given by fitting equation (1) to the p_T spectra of identified species at 2.76 TeV with the least χ^2 s method. The square root of the sum of the statistical and systematic uncertainties of the data points has been taken into account in these fits. Table 1 lists the parameters C_q , q , z_0 as well as their uncertainties returned by the fits. The last line of this table shows the χ^2 s per degrees of freedom (dof), named reduced χ^2 s, for these fits.

Table 1. The parameters C_q , q and z_0 of the scaling functions $\Phi(z)$ for pions, kaons and protons. The uncertainties quoted are due to the statistical plus systematic errors of the data points added in quadrature. The last line shows the reduced χ^2 s for the fits on the p_T spectra of identified particles at 2.76 TeV.

	Pions	Kaons	Protons
C_q	6.01 ± 0.08	0.214 ± 0.002	0.0546 ± 0.0008
q	1.1416 ± 0.0007	1.1402 ± 0.0004	1.115 ± 0.002
z_0 (GeV/c)	0.1308 ± 0.0008	0.193 ± 0.001	0.220 ± 0.002
χ^2/dof	0.49	0.23	0.40

In [4], for pp collisions, the scaling parameters K and A at 0.9 and 2.76 TeV are determined by fitting the scaled Tsallis distribution in equation (2) of that reference to the p_T spectra at these two energies. The quality of the fit heavily depends on the parameters in the scaled Tsallis distribution, C_q , q and z_0 , which are fixed to the values obtained at 7 TeV. Thus K and A at 0.9 and 2.76 TeV rely on the shape of the scaling function in equation (1) of the reference. In this paper, in order to eliminate this reliance, we adopt the quality factor (QF) method to determine K and A for pions, kaons and protons at 0.9 and 7 TeV. In this method, the QF is defined in terms of a set of data points (u^i, v^i) [5, 6]

$$QF(K, A) = \left[\sum_i \frac{(v^i - v^{i-1})^2}{(u^i - u^{i-1})^2 + \varepsilon^2} \right]^{-1}, \quad (2)$$

where $u^i = p_T^i/K$, $v^i = \log(A \cdot (2\pi p_T^i)^{-1} d^2 N^i / dp_T^i dy^i)$, the small constant ε is taken as 0.01 and is utilized to keep the sum being finite when two points have the same u value. Before entering the QF formula, (u^i, v^i) has been rescaled so that $0 \leq u^i, v^i \leq 1$, and u^i are ordered. Obviously, two successive data points being close in u and far in v will give a large contribution to the sum in equation (2). As a result, a set of data points with a small sum (thus a large QF) is expected to lie close to a unique curve. For the scaling parameters at 0.9 (7) TeV, we utilize the data points at 0.9 (7) and 2.76 TeV to determine them. The best set of (K, A) for 0.9 (7) TeV is chosen to be the one which globally maximizes the QF. Table 2 tabulates K and A for pions, kaons and protons in pp collisions at 0.9, 2.76 and 7 TeV. In equation (2), the errors of data points are not taken into account. As described in [5], these errors can be introduced into the QF in the following way:

$$QF(K, A) = \left[\sum_i \frac{(v^i - v^{i-1})^2 \rho^i \rho^{i-1}}{(u^i - u^{i-1})^2 + \varepsilon^2} \right]^{-1}, \quad (3)$$

where ρ^i is the statistical uncertainty of data point i . Obviously, with this definition, the statistical scatter of different data points now have been taken into account in the QF. By maximizing the QF in equation (3), we can get another set of (K, A) . We take the difference between the two sets of (K, A) returned by the maximization of the QFs in equations (2) and (3) as the uncertainties of (K, A) .

Table 2. K and A for pions, kaons and protons at 0.9, 2.76 and 7 TeV. The errors quoted are taken from the difference between the two sets of (K, A) returned by the maximization of the QFs in equations (2) and (3).

	\sqrt{s} (TeV)	K	A
Pions	0.9	0.93 ± 0.04	1.11 ± 0.14
	2.76	1	1
	7	1.06 ± 0.02	0.89 ± 0.07
Kaons	0.9	0.913 ± 0.007	1.05 ± 0.01
	2.76	1	1
	7	1.14 ± 0.04	1.05 ± 0.14
Protons	0.9	0.926 ± 0.001	1.08 ± 0.02
	2.76	1	1
	7	1.108 ± 0.006	1.04 ± 0.02

Since the scaling parameters K and A at 0.9 and 7 TeV now are obtained with the QF method, we could alternatively determine the scaling function $\Phi(z)$ by fitting equation (1) to the combined data points at 0.9, 2.76 and 7 TeV, rather than to the points only at 2.76 TeV. However, the parameters C_q , q and z_0 returned by this fit are consistent with the ones in table 1 when considering their uncertainties. Thus we will use these three parameters in table 1 throughout this work. With K and A in table 2, we plot the scaled pion p_T spectra at 0.9, 2.76 and 7 TeV in terms of the scaling variable z in the upper panel of figure 1. In the log scale, we observe that all data points at different energies now are shifted to the same curve within error bars. This curve is described by the pion scaling function $\Phi(z)$ in equation (1) with parameters in the second column of table 1. In order to see the agreement between the experimental data and the fitted results, we evaluate the ratios, $R = (\text{data} - \text{fitted})/\text{data}$, at 0.9, 2.76 and 7 TeV. The lower panel of figure 1 shows the R distribution as a function of z . It is obvious that the R values of all data points are in the range between -0.2 and 0.2, which implies that the data and fitted curve agreement is within 20%. This agreement roughly corresponds to the systematic error on R and the accuracy of the fit. If we take into account the systematic uncertainties on R , then the accuracy of the fit is around 10%. Considering the fact that the data in the pion p_T spectra cover about 10 orders of magnitude, the fit performed on the pion p_T spectra is good.

The scaling behaviours of the kaon and proton p_T spectra at 0.9, 2.76 and 7 TeV are presented in the upper panels of figures 2 and 3. As done for pions, we show the R distributions for kaons and protons in the lower panels of these two figures. For kaons, except for the last data point at 7 TeV, all the data points and the fitted curve agree within 20%. For protons, the data points are consistent with the curve within 20% when z is smaller than 10 GeV/c. When $z > 10$ GeV/c, the yield and experimental errors for protons at 2.76 TeV have the same order. Thus the values of R are large in this high p_T region.

So far we have seen that the p_T spectra of pions, kaons or protons at 0.9, 2.76 and 7 TeV indeed exhibit a scaling behaviour when these spectra are expressed in terms of z . The scaling function $\Phi(z)$ could be written as the modified Tsallis distribution in equation (1) phenomenologically. As described in [12], the q parameter of the Tsallis distribution for the original p_T spectra of identified particles depends on the collision energy. In this work, the conclusion drawn from the scaling behaviours of identified particles p_T spectra is that the q parameters of the scaled (not the original) p_T spectra at 0.9 and 7 TeV are the same as the one at 2.76 TeV. Thus this conclusion is not in contradiction with the result from [12]. Since the scaling function $\Phi(z)$ depends on the choice of the scaling parameters K and A at 2.76 TeV, we would like to eliminate it by replacing z with $u = z/\langle z \rangle$. The $\langle z \rangle$ values for the pion, kaon and proton p_T spectra are 0.442 ± 0.007 , 0.701 ± 0.008 and 0.83 ± 0.02 GeV/c, where the errors originate from the uncertainties of C_q , q and z_0 . With the substitution of $\langle z \rangle$ and $\Phi(z)$ into $\Psi(u)$ defined in section 2, we can get the normalized scaling functions for pions, kaons and protons as

$$\Psi(u) = C'_q \left[1 - (1 - q') \frac{\sqrt{(m')^2 + u^2 - m'}}{u_0} \right]^{\frac{1}{1-q'}}, \quad (4)$$

where $C'_q = \langle z \rangle^2 C_q / \int_0^\infty \Phi(z) z dz$, $q' = q$, $u_0 = z_0/\langle z \rangle$ and $m' = m/\langle z \rangle$. Table 3 shows the values of C'_q , q' and u_0 for these identified particles.

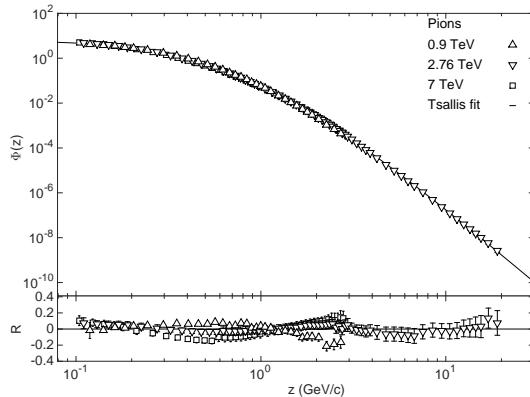


Figure 1. Upper panel: the scaling behaviour of the pion p_T spectra presented in z for pp collisions at 0.9, 2.76 and 7 TeV. The solid curve is described by equation (1) with parameters in the second column of table 1. The data points are taken from [8, 9, 10]. Lower Panel: the R distribution as a function of z for pions.

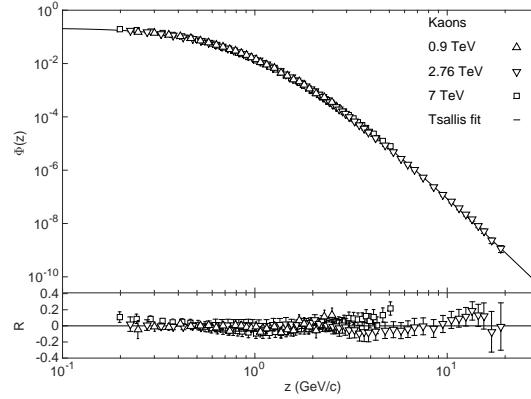


Figure 2. Upper panel: the scaling behaviour of the kaon p_T spectra presented in z for pp collisions at 0.9, 2.76 and 7 TeV. The solid curve is described by equation (1) with parameters in the third column of table 1. The data points are taken from [8, 9, 10]. Lower Panel: the R distribution as a function of z for kaons.

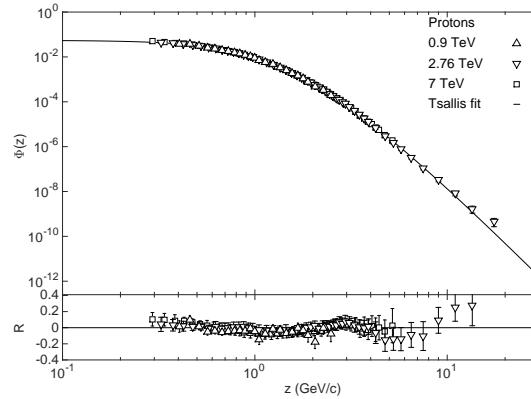


Figure 3. Upper panel: the scaling behaviour of the proton p_T spectra presented in z for pp collisions at 0.9, 2.76 and 7 TeV. The solid curve is described by equation (1) with parameters in the fourth column of table 1. The data points are taken from [8, 9, 10]. Lower Panel: the R distribution as a function of z for protons. The R value for the last data point at 7 TeV is around 0.7 and not shown in the lower panel.

Table 3. The parameters C'_q , q' and u_0 in the normalized scaling functions $\Psi(u)$ for pions, kaons and protons. The uncertainties quoted are due to the errors of parameters C_q , q and z_0 in table 1.

	Pions	Kaons	Protons
C'_q	3.97 ± 0.03	2.86 ± 0.03	2.28 ± 0.03
q'	1.1416 ± 0.0007	1.1402 ± 0.0004	1.115 ± 0.002
u_0	0.296 ± 0.009	0.275 ± 0.004	0.26 ± 0.01

Now we would like to explore the difference among the normalized scaling functions

$\Psi_{\text{Pions}}(u)$, $\Psi_{\text{Kaons}}(u)$ and $\Psi_{\text{Protons}}(u)$ for pions, kaons and protons (see the upper panel of figure 4). In order to illustrate this difference clearly, we present the distribution of $r = \Psi_{\text{Kaons(Protons)}}(u)/\Psi_{\text{Pions}}(u)$ in the lower panel of figure 4. In the small (large) p_T region with $u < 0.6$ ($u > 2$), $\Psi_{\text{Pions}}(u)$ is larger than $\Psi_{\text{Protons}}(u)$ and $\Psi_{\text{Kaons}}(u)$, and the discrepancy goes down (up) with the increase of u . While in the moderate p_T region with $0.6 < u < 2$, $\Psi_{\text{Pions}}(u)$ is smaller than $\Psi_{\text{Protons}}(u)$ and $\Psi_{\text{Kaons}}(u)$. These differences could be validated by the comparison among the normalized moments of the transverse momentum distributions $\langle p_T^n \rangle / \langle p_T \rangle^n$ for pions, kaons and protons. This normalized moment is expressed in terms of $\Psi(u)$, $\langle p_T^n \rangle / \langle p_T \rangle^n = \int_0^\infty u^n \Psi(u) u du$. Table 4 gives $\langle p_T^n \rangle / \langle p_T \rangle^n$ for the identified particles with $n = 2, 3, 4$. Because of $\Psi_{\text{Pions}}(u)$ being above $\Psi_{\text{Protons}}(u)$ and $\Psi_{\text{Kaons}}(u)$ at small (large) u , the normalized moment of the pion p_T distribution is larger than the ones of the proton and kaon p_T distributions for small (large) n . Identical results are obtained from the comparison between the normalized moments of pions and protons produced in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV [3].

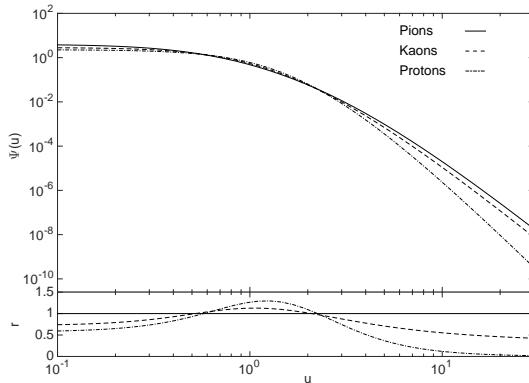


Figure 4. Upper panel: the normalized scaling functions for pions, kaons and protons. Lower Panel: the distribution of the ratio between the normalized scaling functions of kaons (protons) and pions.

Table 4. The normalized moments of the transverse momentum distributions $\langle p_T^n \rangle / \langle p_T \rangle^n$ for pions, kaons and protons. The uncertainties quoted are due to the errors of C'_q , q' and u_0 in table 3.

n	Pions	Kaons	Protons
2	1.9 ± 0.2	1.7 ± 0.1	1.5 ± 0.2
3	6.5 ± 1.0	4.9 ± 0.3	3.3 ± 0.5
4	44.8 ± 8.2	26.8 ± 2.2	10.7 ± 2.2

4. Comparison to the scaling behaviour presented in τ

In [7], a geometrical scaling behaviour appeared when the p_T spectra of identified particles in pp collisions at 0.9, 2.76 and 7 TeV were presented in terms of the scaling variable $\tau = p_T/Q_0(p_T/\sqrt{s})^{\lambda/2}$, where $Q_0 = 1$ GeV/c and $\lambda \approx 0.27$. For the sake of illustrating this scaling behaviour, the author in that article chose the pion p_T spectra at 7 TeV as a reference, and showed the ratio between the spectra at 7 and 0.9 (2.76) TeV as a function of τ . In this work, in order to compare the scaling behaviours presented in z and τ directly, we select the pion spectra at 2.76 TeV as a baseline, and plot the spectra at 0.9, 2.76 and 7 TeV as a function of τ in the upper panel of figure 5. The pion spectra at 2.76 TeV is described by the modified Tsallis fit in equation (1) with z replaced by τ . The lower panel of figure 5 shows the R distribution as a function of τ . The data points at 0.9, 2.76 and 7 TeV agree with the fitted curve within an accuracy of 20%. This accuracy is comparable to the one of the scaling behaviour presented in z . Analogous results are obtained in the comparison between the scaling behaviours presented in z and τ for the kaon and proton p_T spectra. However, the author in [7] showed that when $\tau > 6$, the geometrical scaling starts to break down. The data at 7 TeV utilized in that work are preliminary and have not been published by the ALICE collaboration so far. In order to see whether there is a breaking in the scaling behaviour presented in z , more public data in the high p_T region at 7 TeV are needed.

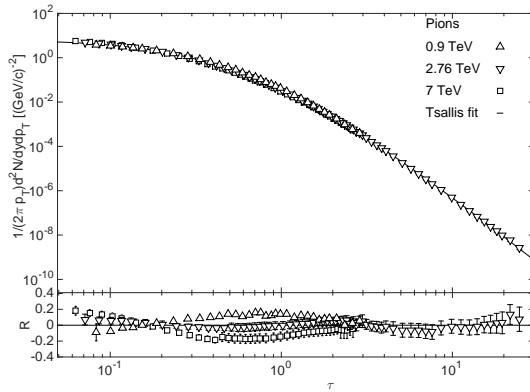


Figure 5. Upper panel: the geometrical scaling of the pion p_T spectra presented in τ for pp collisions at 0.9, 2.76 and 7 TeV. The solid curve is described by the pion scaling function in equation (1) with z replaced by τ . The data points are taken from [8, 9, 10]. Lower Panel: the R distribution as a function of τ for pions.

5. Colour String Percolation Model

In [7, 13], the author suggested the scaling behaviour presented in τ could be understood by the gluon saturation mechanism. However, in this work, we will argue that the scaling behaviour presented in z for the identified particles could be explained by the colour string percolation (CSP) model [14, 15] in a quantitative way simultaneously.

In this model, colour strings are stretched between the partons of protons in pp collisions. These strings then decay into new ones with the emission of $q\bar{q}$ pairs. Identified hadrons are produced through the hadronization of these new strings. The strings are viewed as small areas of $S_1 = \pi r_0^2$ with $r_0 \approx 0.2$ fm in the transverse plane. The number of strings grows with the increase of the collision energy. When there are n strings, they start to overlap and form clusters with transverse areas of S_n . The mean transverse momentum squared $\langle p_T^2 \rangle_{ni}$ of identified particles produced by a cluster can be written as $\langle p_T^2 \rangle_{ni} = \sqrt{nS_1/S_n} \langle p_T^2 \rangle_{1i}$, where $\langle p_T^2 \rangle_{1i}$ is the mean p_T^2 of identified particles produced by a single string, and nS_1/S_n is the degree of string overlap. If strings just get in touch with each other, then $S_n = nS_1$ and $nS_1/S_n = 1$. If strings maximally overlap with each other, then $S_n = S_1$ and $nS_1/S_n = n$ with $n > 1$. The p_T spectra of identified particles produced in pp collisions is deemed as a superposition of the p_T spectra produced by each cluster, $f(x, p_T)$, weighted with the distribution of the cluster's size, $W(x)$. Here the cluster size x is equivalent to the inverse of $\langle p_T^2 \rangle_{ni}$. In [14] and [15], the fragmentation function for the cluster is chosen as the Schwinger formula $f(x, p_T) = \exp(-p_T^2 x)$ [16]. However, this function only describes the spectra well at the soft p_T region. In order to describe the spectra at the high p_T region as well, the Schwinger formula should be replaced by $f(x, p_T) = \exp(-\sqrt{2x} p_T)$ [17, 18]. $W(x)$ is supposed to be a gamma distribution, $W(x) = \frac{\gamma}{\Gamma(k)} (\gamma x)^{k-1} \exp(-\gamma x)$, where γ and k are free parameters. Thus the p_T distribution of identified particles in pp collisions is

$$\frac{d^2N}{2\pi p_T dp_T dy} = C \int_0^\infty W(x) f(x, p_T) dx, \quad (5)$$

where C is a normalization constant for the total number of clusters formed for identified particles before hadronization. In order to see whether the CSP model can describe the scaling behaviour presented in z , we fit equation (5) to the p_T spectra of identified particles at 2.76 TeV. The parameters C , r and k returned by the fits as well as the reduced χ^2 's are tabulated in table 5. As shown in the upper panel of figure 6, the data points for pions agree with the CSP fit well in log scale. The accuracy of this agreement is around 20%, which can be seen from the R distribution in the lower panel of the figure. Similar results are obtained for kaons and protons. As the k parameters are different for the identified particles, pions, kaons and protons originate from different distributions of clusters.

Table 5. The parameters C , r and k of the CSP fits on the spectra of pions, kaons and protons at 2.76 TeV. The uncertainties quoted are due to the statistical plus systematic errors of the data points added in quadrature. The last line shows the reduced χ^2 's for the fits.

	Pions	Kaons	Protons
C	9.1 ± 0.2	0.53 ± 0.01	0.160 ± 0.007
γ	0.140 ± 0.003	0.342 ± 0.008	0.91 ± 0.07
k	3.15 ± 0.02	3.33 ± 0.02	5.0 ± 0.2
χ^2/dof	1.53	0.91	1.91

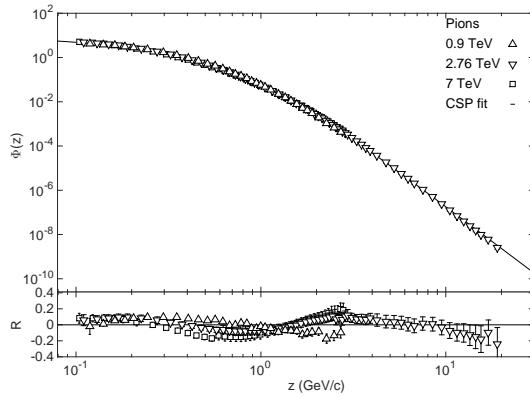


Figure 6. Upper panel: the scaling behaviour of the pion p_T spectra presented in z for pp collisions at 0.9, 2.76 and 7 TeV. The solid curve is described by the CSP fit in equation (5) with parameters in the second column of table 5. The data points are taken from [8, 9, 10]. Lower Panel: the R distribution as a function of z for pions.

Now we would like to seek for the reason why the CSP model can describe the scaling behaviour of the identified p_T spectra. Under the transformation $x \rightarrow x' = \lambda x$, $\gamma \rightarrow \gamma' = \gamma/\lambda$ and $p_T \rightarrow p'_T = p_T/\sqrt{\lambda}$, both $W(x)$ and $f(x)$ are invariant. So the p_T spectra of identified particles in equation (5) are also invariant. This invariance is the scaling behaviour we are looking for. Comparing the p'_T transformation in the CSP model $p'_T \rightarrow p'_T \sqrt{\lambda}$ with the one used to search for the scaling behaviour $p_T \rightarrow p_T/K$, we know that K is proportional to $1/\sqrt{\lambda}$. As described in [15, 18], $\lambda = \langle S_n/nS_1 \rangle^{1/2}$, where the average is taken over all clusters for identified particles, thus K should be proportional to $\langle nS_1/S_n \rangle^{1/4}$. Since the degree of string overlap nS_1/S_n grows with energy, the scaling parameter K should also increase with energy. That's indeed what we see for the identified particles in the third column of table 2. Therefore the CSP model can explain the scaling behaviours presented in z for pions, kaons and protons separately. However, the rate of the K values increasing with energy for pions is different with the one for kaons or protons. This could also be explained by the CSP model. As $K = \langle p_T \rangle / \langle z \rangle$, and the values of $\langle z \rangle$ are the same for pions (kaons or protons) at 0.9,

2.76 and 7 TeV, the ratio between the values of K at different energies should be equal to the ratio between the values of $\langle p_T \rangle$ at different energies. $\langle p_T \rangle$ is evaluated in terms of the CSP model as [7]

$$\langle p_T \rangle = \frac{\int_0^\infty \int_0^\infty W(x) f(x, p_T) p_T^2 dx dp_T}{\int_0^\infty \int_0^\infty W(x) f(x, p_T) p_T dx dp_T}. \quad (6)$$

The cluster fragmentation functions $f(x, p_T)$ are the same for pions (kaons or protons) at different energies. However, the cluster's size distributions $W(x)$ are different for different particles, which can be seen from the comparison among the k values for pions, kaons and protons in table 5. As a result, for different species of particles, the values of K change in a different rate with energy. In a summary, the CSP model could be utilized to explain the scaling behaviours presented in z for the identified particles simultaneously.

6. Conclusions

In this paper, we have extended the scaling behaviour in the inclusive charged hadron p_T spectra to the spectra of identified particles at 0.9, 2.76 and 7 TeV. This scaling behaviour is exhibited when p_T is replaced by $z = p_T/K$. The scaling parameter K is determined by the quality factor method, which does not rely on the shape of the scaling function. We have argued that the pions, kaons and protons originate from different distributions of clusters formed by strings overlapping, and the scaling behaviours of these identified particles could be explained by the colour string percolation model in a quantitative way at the same time.

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